Numerical modelling of freezing and thawing process of packed food product

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Abstract. One of the basic methods of prolonging the durability of food products with a high water content is their freezing. An important parameter used in the design of freezer facilities is the freezing time. To determine this parameter the methods of numerical modeling can be used. In this paper the mathematical model of freezing and thawing process of packed food is presented. The problem has been solved using the finite difference method. The authorial program allows one to determine the temperature distribution in the domain considered, the freezing time etc. In the final part the example of computations is shown.

FORMULATION OF THE PROBLEM

Assuming heat transfer in one dominant direction (Figure 1), the Fourier equation has the following form

$$0 < x < L: \quad c(T)\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T(x,t)}{\partial x}\right] + Q(x,t) \tag{1}$$

where $c(T)$ is volumetric specific heat of food product, T denotes the temperature, x is a spatial coordinate, t is the time, $Q(x, t)$ is the source function related to the freezing (thawing) process.

The thermal conductivity $\lambda(T)$ is defined as follows

$$\lambda(T) = \begin{cases} \lambda_N, & T > T_1 \\ \dfrac{\lambda_N + \lambda_F}{2}, & T_2 \le T \le T_1 \\ \lambda_F, & T < T_2 \end{cases} \tag{2}$$

where the temperatures T_1, T_2 correspond to the beginning and the end of the freezing (thawing) process, respectively, λ_N, λ_F are the constant thermal conductivities of natural and frozen state.

It should be noted that the freezing (thawing) process is taken into account by an introduction of the parameter called a substitute thermal capacity, while the approach proposed is known as the fixed domain method [1].

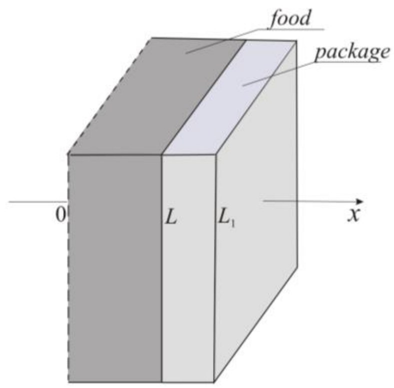

FIGURE 1. Domain considered

The temperature field in the package is described by following equation

$$L < x < L_1: \quad c_p \frac{\partial T_p(x,t)}{\partial t} = \lambda_p \frac{\partial^2 T_p(x,t)}{\partial x^2} \tag{3}$$

where λ_p and c_p are the thermal conductivity and volumetric specific heat of the material from which the packaging is made. The condition determining the heat exchange on the surface $x = L_1$ of the package and a cooling or warming environment has the form of Robin condition

$$q_p(x,t) = -\lambda_p \frac{\partial T_p(x,t)}{\partial x} = \alpha \left[T_p(x,t) - T_f \right] \tag{4}$$

where α is the heat transfer coefficient and T_f is the cooling or warming temperature.
Assuming that the product (as a plate) is evenly cooled on both sides, only half of the product is considered and then

$$x = 0: \quad q(x,t) = \lambda(T) \frac{\partial T(x,t)}{\partial x} = 0 \tag{5}$$

Between the food product and the package the ideal thermal contact is assumed

$$x = L: \begin{cases} T(x, t) = T_p(x, t) \\ q(x, t) = q_p(x, t) \end{cases} \tag{6}$$

The initial temperature T_0 of the food product and package is also known

$$t = 0: \quad T(x,0) = T_0, \quad T_p(x,0) = T_0 \tag{7}$$

The problem has been solved using the explicit scheme of finite difference method [2].

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REFERENCES

1. B. Mochnacki, E. Majchrzak, G. Kałuża, Modelling of biological tissue freezing process with regard to sensitivity analysis, *Acta of Bioengineering and Biomechanics,* 6, 1, pp. 480-484, (2004).
2. B. Mochnacki and J.S. Suchy, *Symulacja i modelowanie krzepnięcia odlewów*, (WNT, Warszawa, 1993).